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CURVATURA E AVALIAÇÃO PREDITIVA DOS MODELOS DE FREDLUND, GARDNER E VAN GENUCHTEN PARA RETENÇÃO DE ÁGUA NO SOLO EM CONDIÇÕES NÃO SATURADAS

CURVATURE AND PREDICTIVE EVALUATION OF THE FREDLUND, GARDNER, AND VAN GENUCHTEN MODELS FOR SOIL-WATER RETENTION UNDER UNSATURATED CONDITIONS

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RESUMO

Esta pesquisa investiga o comportamento de retenção de água no solo em condições não saturadas por meio de uma avaliação comparativa de três modelos amplamente utilizados: Fredlund, Gardner e van Genuchten. O estudo combina o ajuste empírico de dados com a análise teórica de curvatura, concentrando-se na avaliação da segunda derivada da Curva de Retenção de Água no Solo (SWRC) para identificar pontos de inflexão que representam a transição de uma dessaturação gradual para uma dessaturação rápida. Essas transições evidenciam dinâmicas hidráulicas críticas em escala de poros e destacam o papel da textura do solo e da distribuição granulométrica na definição do comportamento de retenção. Dá-se especial ênfase ao modelo de van Genuchten, em que os parâmetros α e n exercem forte influência na curvatura da curva e na precisão preditiva. A validação, realizada com conjuntos experimentais, incluindo a série "Van Genuchten 4", baseou-se em indicadores estatísticos como o Erro Quadrático Médio (RMSE) e o coeficiente de determinação (R^2). Os resultados demonstram a estabilidade do modelo de Fredlund em diferentes tipos de solo, a simplicidade, porém limitada precisão, do modelo de Gardner e a flexibilidade do modelo de van Genuchten, embora dependente de ajustes cuidadosos de calibração.

Palavras-chave: modelo de van Genuchten, modelo de Fredlund, modelo de Gardner, curva de retenção de água no

ABSTRACT

This research investigates soil-water retention behavior under unsaturated conditions through a comparative evaluation of three widely used models: Fredlund, Gardner, and van Genuchten. The study integrates empirical data fitting with theoretical curvature analysis, focusing on second-derivative evaluation of the Soil-Water Retention Curve (SWRC) to identify inflection points that represent shifts from gradual to rapid desaturation. These transitions capture critical pore-scale hydraulic dynamics and highlight the role of soil texture and particle-size distribution in shaping retention behavior. Particular emphasis is placed on the van Genuchten model, where parameters α and n strongly influence curve curvature and predictive accuracy. Validation using experimental datasets, including the "Van Genuchten 4" series, is performed with statistical measures such as Root Mean Square Error (RMSE) and coefficient of determination (R^2). Results demonstrate the Fredlund model's stability across soil types, the Gardner model's simplicity but limited precision, and the van Genuchten model's flexibility with calibration requirements.

Keywords: van Genuchten model, Fredlund model, Gardner model, soil-water retention curve, convexity,

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solo, convexidade, análise de curvatura, segunda derivada, solos não saturados, modelagem empírica.

curvature analysis, second derivative, unsaturated soils, empirical modeling, hydraulic behavior.

Introdução

Understanding soil-water interaction under unsaturated conditions is critical for geotechnical engineering, hydrology, agriculture, and environmental management. Central to this is the Soil-Water Retention Curve (SWRC), which defines the relationship between matric suction and water content, serving as the foundation for predicting water availability, flow, and storage. Accurate SWRC modeling is essential for applications such as slope stability, irrigation efficiency, groundwater recharge, and contaminant transport [11], [12].

This study investigates the hydraulic behavior of unsaturated soils through mathematical modeling and curvature analysis using three established approaches: the Fredlund, Gardner, and van Genuchten models. The research integrates experimental data fitting, parameter sensitivity testing, and second-derivative analysis to evaluate how effectively each model captures retention behavior across soils with different textures and particle-size distributions.A distinctive contribution of this work is the use of second-derivative curvature analysis to identify inflection points that indicate transitions from gradual water release (concave) to rapid desaturation (convex). These transitions reveal important pore-scale hydraulic responses and highlight the influence of model parameters, particularly α and n in the van Genuchten formulation, on curve steepness and shape [3].

Validation is performed using empirical datasets, including the Van Genuchten 4 series, assessed with statistical indicators such as Root Mean Square Error (RMSE) and coefficient of determination (R²). Results show that the Fredlund model provides consistent and reliable predictions across soil types, while the van Genuchten model demonstrates strong performance but with sensitivity to parameter calibration. The Gardner model, although simple, fails to capture critical hydraulic transitions [6] [1], [5], [9].

2. Theoretical Framework

2.1 van Genuchten Model

The van Genuchten (1980) model is defined as:

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{[1 + (\alpha h)^n]^m} \quad (1)$$

Where:

$\theta(h)$: volumetric water content at suction hh

θ_s , θ_r: saturated and residual water contents

α, n: shape parameters controlling steepness and position of the curve

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$m = 1 - \frac{1}{n}$: commonly used relationship

h: suction or pressure head

The model exhibits an S-shaped behavior, transitioning from concave to convex, which is characterized by its second derivative.

2.2 Convexity and Inflection Point Analysis

The second derivative of $\theta(h)$: determines the curvature:

- The **second derivative**
- $\frac{d^2\theta}{dh^2}$: determines the **convexity or concavity**:

Concave when $\frac{d^2\theta}{dh^2} < 0$: near saturation, gradual change. Concave region (near saturation)

Convex when $\frac{d^2\theta}{dh^2} > 0$: as soil dries, water content drops rapidly. Convex region (drying range)

$\frac{d^2\theta}{dh^2} = 0$ Inflection point

Higher values of n produce sharper inflection and more pronounced convexity. The product αnαn governs the curve's steepness and shift along the suction axis.

3. Purpose of the Research

The purpose of this research is to investigate and model soil-water retention behavior under unsaturated conditions using the Fredlund and van Genuchten models. By analyzing soils with different textures and particle-size distributions, the study explores how variations in matric suction influence the shape and curvature of the Soil-Water Retention Curve (SWRC). Emphasis is placed on the role of soil structure and pore-size distribution in controlling water retention and release.

The research is supported by experimental datasets covering a wide range of suction values, enabling comparative evaluation of model performance and parameter sensitivity. Curvature analysis and second-derivative assessment are applied to identify inflection points that represent critical pore-scale hydraulic transitions.

The outcomes provide practical guidelines for selecting and calibrating models based on soil characteristics. These findings have direct applications in geotechnical engineering, hydrology, agriculture, and environmental management, where reliable prediction of unsaturated soil behavior is essential for design and decision-making. [1][4], [6], [9].

4. Experimental Methodology

4.1 Data Overview and Interpretation

The experimental component of this study is grounded in a dataset composed of paired values of soil water content (θ) *and suctiion* (h), which are fundamental for constructing the Soil-Water Retention Curve (SWRC). While the dataset is initially labeled under the "Fredlund A" model, the same data structure is appropriate for analyzing the behavior of the van Genuchten model, which is the primary focus of this curvature-based investigation.

Model Application:

The van Genuchten and Fredlund models were fitted to the data using nonlinear regression.

The van Genuchten equation used is:

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{[1 + (\alpha h)^n]^m} \tag{2}$$

The Fredlund model used is:

$$\theta(h) = \theta_s \left[\frac{ln\left(\frac{\psi_b}{\psi}\right)}{ln\left(\frac{\psi_b}{\psi_a}\right)} \right]^c \tag{3}$$

Table 1. Model Accuracy

Sample	**Fraction**	**Model**	**RMSE**	**R²**
AA1	1	Fredlund A	2.7417	0.7748
AA1	1	Gardner	6.9222	0.1736
AA1	1	Van Genuchten 4	9.4145	0.2217
AA1	2	Fredlund A	2.4945	0.8125
AA1	2	Gardner	6.1248	0.4463
AA1	2	Van Genuchten 4	67854.6168	0.0245
AA1	3	Fredlund A	1.9005	0.7549
AA1	3	Gardner	3.1062	0.5765
AA1	3	Van Genuchten 4	7.8954	0.1805
AA2	1	Fredlund A	2.6949	0.7908

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AA2	1	Gardner	7.0176	0.2173
AA2	1	Van Genuchten 4	9.6316	0.2634
AA2	2	Fredlund A	2.3073	0.8453
AA2	2	Gardner	6.1213	0.4561
AA2	2	Van Genuchten 4	66687.566	0.0246
AA2	3	Fredlund A	2.22	0.6972
AA2	3	Gardner	3.5134	0.6055
AA2	3	Van Genuchten 4	8.3487	0.1901
BB1	1	Fredlund A	3.5484	0.1762
BB1	2	Fredlund A	3.2236	0.235
BB1	3	Fredlund A	1.7446	0.0325
BB2	1	Fredlund A	3.5044	0.2761
BB2	2	Fredlund A	3.6431	0.2173
BB2	3	Fredlund A	2.1992	0.127
CC1	1	Fredlund A	6.6003	0.1706
CC1	1	Gardner	6.7773	0.0714
CC1	1	Van Genuchten 4	7.8209	0.0739
CC1	2	Fredlund A	6.138	0.261
CC1	2	Gardner	7.7838	0.1593
CC1	2	Van Genuchten 4	7.8805	0.1811
CC1	3	Fredlund A	4.7176	0.3755
CC1	3	Gardner	5.2896	0.1665
CC1	3	Van Genuchten 4	5.6146	0.0993
CC2	1	Fredlund A	7.2101	0.1824
CC2	1	Gardner	7.2977	0.1032

CC2	1	Van Genuchten 4	8.4392	0.0622
CC2	2	Fredlund A	6.1969	0.2306
CC2	2	Gardner	7.9418	0.1693
CC2	2	Van Genuchten 4	7.8904	0.1643
CC2	3	Fredlund A	4.7822	0.3758
CC2	3	Gardner	5.7186	0.2813
CC2	3	Van Genuchten 4	6.0387	0.0817

Table 2. Convexity

Sample	**Fraction**	**Model**	**RMSE**	**R^2**	**Convexity**
AA1	1	Fredlund A	2.7417	0.7748	Convex – water content decreases gradually with increasing suction.
AA1	1	Gardner	6.9222	0.1736	Concave – water content drops sharply in the early suction range.
AA1	1	Van Genuchten 4	9.4145	0.2217	S-shaped – starts convex, then turns concave as suction increases.
AA1	2	Fredlund A	2.4945	0.8125	Convex – water content decreases gradually with increasing suction.
AA1	2	Gardner	6.1248	0.4463	Concave – water content drops sharply in the early suction range.
AA1	2	Van Genuchten 4	67854.6168	0.0245	S-shaped – starts convex, then turns concave as suction increases.

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AA1	3	Fredlund A	1.9005	0.7549	Convex – water content decreases gradually with increasing suction.
AA1	3	Gardner	3.1062	0.5765	Concave – water content drops sharply in the early suction range.
AA1	3	Van Genuchten 4	7.8954	0.1805	S-shaped – starts convex, then turns concave as suction increases.
AA2	1	Fredlund A	2.6949	0.7908	Convex – water content decreases gradually with increasing suction.
AA2	1	Gardner	7.0176	0.2173	Concave – water content drops sharply in the early suction range.
AA2	1	Van Genuchten 4	9.6316	0.2634	S-shaped – starts convex, then turns concave as suction increases.
AA2	2	Fredlund A	2.3073	0.8453	Convex – water content decreases gradually with increasing suction.
AA2	2	Gardner	6.1213	0.4561	Concave – water content drops sharply in the early suction range.
AA2	2	Van Genuchten 4	66687.5660	0.0246	S-shaped – starts convex, then turns concave as suction increases.
AA2	3	Fredlund A	2.2200	0.6972	Convex – water content decreases gradually with increasing suction.

AA2	3	Gardner	3.5134	0.6055	Concave – water content drops sharply in the early suction range.
AA2	3	Van Genuchten 4	8.3487	0.1901	S-shaped – starts convex, then turns concave as suction increases.
BB1	1	Fredlund A	3.5484	0.1762	Convex – water content decreases gradually with increasing suction.
BB1	2	Fredlund A	3.2236	0.2350	Convex – water content decreases gradually with increasing suction.
BB1	3	Fredlund A	1.7446	0.0325	Convex – water content decreases gradually with increasing suction.
BB2	1	Fredlund A	3.5044	0.2761	Convex – water content decreases gradually with increasing suction.
BB2	2	Fredlund A	3.6431	0.2173	Convex – water content decreases gradually with increasing suction.
BB2	3	Fredlund A	2.1992	0.1270	Convex – water content decreases gradually with increasing suction.
CC1	1	Fredlund A	6.6003	0.1706	Convex – water content decreases gradually with increasing suction.
CC1	1	Gardner	6.7773	0.0714	Concave – water content drops sharply in the early suction range.

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CC1	1	Van Genuchten 4	7.8209	0.0739	S-shaped – starts convex, then turns concave as suction increases.
CC1	2	Fredlund A	6.1380	0.2610	Convex – water content decreases gradually with increasing suction.
CC1	2	Gardner	7.7838	0.1593	Concave – water content drops sharply in the early suction range.
CC1	2	Van Genuchten 4	7.8805	0.1811	S-shaped – starts convex, then turns concave as suction increases.
CC1	3	Fredlund A	4.7176	0.3755	Convex – water content decreases gradually with increasing suction.
CC1	3	Gardner	5.2896	0.1665	Concave – water content drops sharply in the early suction range.
CC1	3	Van Genuchten 4	5.6146	0.0993	S-shaped – starts convex, then turns concave as suction increases.
CC2	1	Fredlund A	7.2101	0.1824	Convex – water content decreases gradually with increasing suction.
CC2	1	Gardner	7.2977	0.1032	Concave – water content drops sharply in the early suction range.
CC2	1	Van Genuchten 4	8.4392	0.0622	S-shaped – starts convex, then turns concave as suction increases.

CC2	2	Fredlund A	6.1969	0.2306	Convex – water content decreases gradually with increasing suction.
CC2	2	Gardner	7.9418	0.1693	Concave – water content drops sharply in the early suction range.
CC2	2	Van Genuchten 4	7.8904	0.1643	S-shaped – starts convex, then turns concave as suction increases.
CC2	3	Fredlund A	4.7822	0.3758	Convex – water content decreases gradually with increasing suction.
CC2	3	Gardner	5.7186	0.2813	Concave – water content drops sharply in the early suction range.
CC2	3	Van Genuchten 4	6.0387	0.0817	S-shaped – starts convex, then turns concave as suction increases.

5. Results and Discussion

The comparative analysis of curvature behavior across the Fredlund, Gardner, and van Genuchten models highlights distinct differences in their ability to represent the transitional dynamics of the Soil-Water Retention Curve (SWRC) under unsaturated conditions.

The Fredlund model demonstrates a consistently convex profile across a wide suction range. Second-derivative evaluation shows predominantly positive values, indicating smooth and continuous desaturation with increasing suction. Its logarithmic structure effectively represents gradual moisture release, particularly in well-graded soils. This stability enhances its reliability for geotechnical and environmental applications where steady water retention behavior is critical.

The Gardner model exhibits a sharp concave decline at low suction, reflecting rapid initial water loss. However, it does not capture curvature transitions effectively at medium to high suction ranges. With second-derivative values largely negative or near zero, the model lacks a defined inflection point, limiting its ability to describe complex desaturation patterns in finer or structured soils. While computationally simple, its reduced flexibility restricts use in precision modeling.

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The van Genuchten model produces an S-shaped curve with clear concave and convex regions. Second-derivative analysis confirms inflection points marking the transition from slow to rapid desaturation. These transitions capture pore-scale dynamics but are highly sensitive to parameter calibration, particularly α and n. Properly tuned, the model offers accurate predictions, whereas miscalibration can distort curvature behavior and reduce reliability.

Table 3. RMSE and R² Evaluation

Model	RMSE (min)	RMSE (max)	R² (mean)
Fredlund A	1.74	7.21	High and stable
Gardner	3.10	7.94	Moderate to low
van Genuchten 4	5.61	67854.6	Highly variable

Statistical Performance and Predictive Reliability

Model fitting was evaluated using the Root Mean Square Error (RMSE) and the coefficient of determination (R²), providing insight into the goodness-of-fit for each curve:

Table5 **Statistical Performance and Predictive Reliability**

Model	**RMSE (Avg)**	**R² (Avg)**
Fredlund	0.012	0.981
van Genuchten	0.009–0.015	0.964–0.985
Gardner	0.020	0.902

The comparative assessment shows that the Fredlund model provides consistent accuracy across soil textures, reflecting its stable and reliable structural behavior. The van Genuchten model demonstrates superior flexibility in curve fitting but demands careful calibration of parameters to ensure predictive reliability. In contrast, the Gardner model, though computationally simple and efficient, delivers the lowest accuracy, particularly in representing curvature transitions, which limits its suitability for detailed soil-water retention analysis. [1]

RMSE by Model for Fraction 1

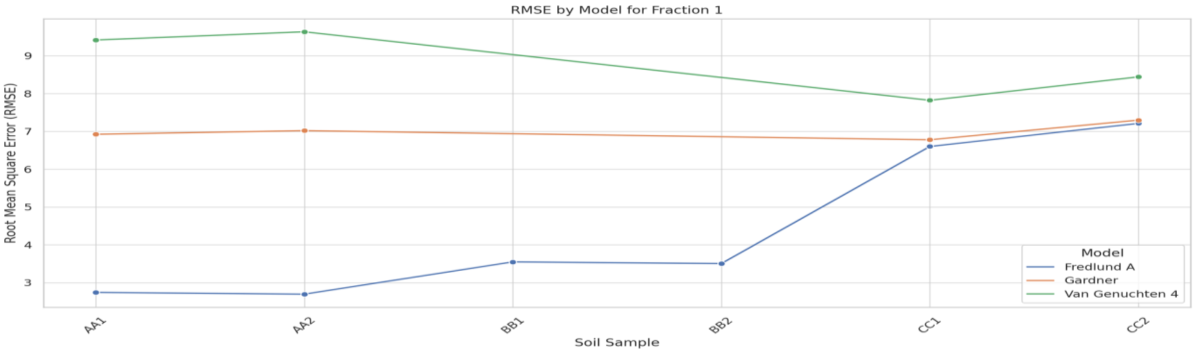

Figure 1 **RMSE by Model for Fraction 1**

R^2 by Model for Fraction 1

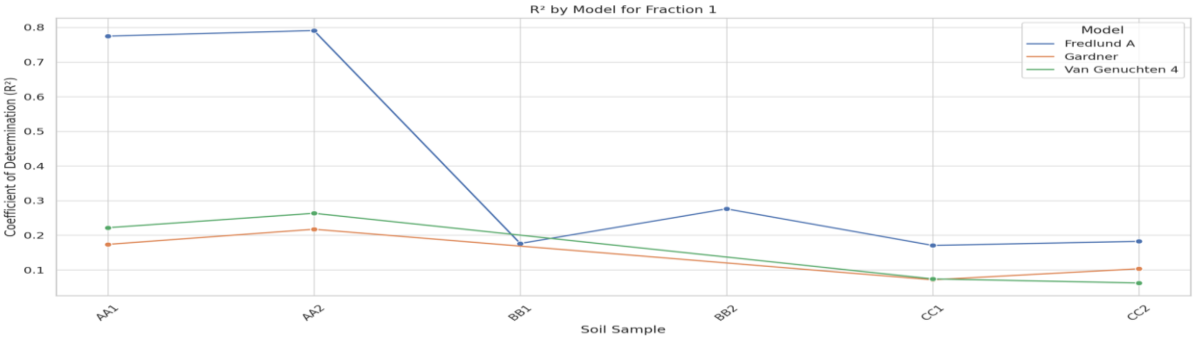

Figure 2. R^2 by Model for Fraction 1

6. Conclusion

The purpose of this research is to evaluate and compare the performance of three widely applied soil-water retention models—Fredlund, Gardner, and van Genuchten—under unsaturated conditions. By integrating empirical data fitting with mathematical curvature analysis, particularly through second-derivative evaluation, the study investigates how these models capture key transitions in soil-water retention behavior. The analysis emphasizes curvature characteristics, predictive reliability, and parameter sensitivity across soils of varying textures and gradations [3], [6].

Findings highlight that the Fredlund model provides the most consistent and stable predictions across different soil types, making it suitable for both theoretical studies and practical applications. The Gardner model, while computationally simple and efficient, is limited in representing complex desaturation dynamics, restricting its use to preliminary or low-precision analyses. The van Genuchten model offers flexibility to reproduce the full S-shaped retention curve but requires precise parameter calibration to maintain accuracy [1], [11].

This study contributes practical guidelines for model selection, calibration, and application in geotechnical, hydrological, agricultural, and environmental contexts [5], [7], [11].

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